

MULTIDIMENSIONAL WELL-  
BEING INEQUALITY SOCIAL  
EVALUATION GINI FUNCTION  
FOR COLOMBIA 2012

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# MULTIDIMENSIONAL WELL-BEING INEQUALITY SOCIAL EVALUATION GINI FUNCTION FOR COLOMBIA 2012

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## 1. ABSTRACT

The traditional literature on inequality measurements focused, for a long time, on achievements represented by one vector of distribution. Current researches consider that individual well-being consist in a multi vector distribution. In that sense, this paper describes the axiomatic framework and calculates the Multidimensional Social Evaluation Gini Function of Decancq and Lugo (2008). It assumes the distribution of capabilities as a complementary road to understand that concept. The empirical measure uses household data from Colombia for 2012 from the National Household Survey (GEIH) over three dimensions of well-being: health, education and income. The results show a Gini index rank between 0.1572 and 0.3632, depending on the bottom sensitivity. In both cases, the current measure is below the traditional Gini evaluated for 2012 (0.539). In addition, calculations shows profound discrepancies in terms of regions.

## 2. INTRODUCTION

The high level of inequality between persons or households in terms of income or wealth is one of the most important problems of Colombia, as well as in most Latin American countries. Considering the country entered recently in the upper middle incomes group, its distribution is not an aspect to be proud<sup>3</sup>. The Gini coefficient<sup>4</sup> for Colombia in 2012

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<sup>3</sup> Colombia's inequality has the 11th position worldwide, according to the World Bank.

<sup>4</sup> The Gini coefficient measures the average difference between pairs of incomes in a distribution, relative to the distribution's mean. It reflects inequality in the income

was 0.539<sup>5</sup>. However, it shows a better off position since 2002, where it was 0.572 (DANE, 2012).

Historically, the research in inequality measures focused on achievements of countries or societies represented by one vector of distribution, using the cumulative distribute function, the Lorenz graphic representation and a variety of positive indices commonly used as Kuznets, Sen, Gini, family of Atkinson and generalized entropy measures. Measurement also moved to a normative way of understand the topic, which is concerned with economic welfare theory, as Dalton or Atkinson's indices.

After Rawls (1971), Sen (1973, 1985) and the Human Development Report of 1990 (HDR), well-being becomes a multidimensional concept considering more variables than merely income, wealth or utility<sup>6</sup>. Accordingly, Sen's concept of distribution incorporates well-being concepts, where it depends on the capability to achieve valuable functions (Walsh, 1995). A more complex and diversified space comes to the analysis and includes others dimensions as health and nutrition, low education and skills, inadequate livelihoods, bad housing conditions, social exclusion and lack of participation.

Current papers and studies of multidimensional inequality approaches originally base their concepts on Kolm's work "Multidimensional egalitarianism" (1977) and Atkinson and Bourguignons' paper "The comparison of multi-dimensional distributions of economic status" (1982). Specifically, the paper "Measuring inequality of Well-being" of Decancq and Lugo bases its methodology on the multidimensional inequality concepts of Atkinson (1970), Sen (1973) and Kolm (1977) in order to derive a multidimensional social evaluation function and the relative multidimensional single parameter Gini inequality measure (Decancq, 2008).

The above ideas mentioned give way to the possibility of extent from the one-dimensional inequality concept to the multidimensional and applying that methodology in the special case of Colombia. The

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dimension. Inside the Gini, a value close to one means that the group of reference has a high inequality between observations, and a value close to zero means that observations are equally distributed.

<sup>5</sup> It means the loss of welfare because of inequality in the distribution of incomes in Colombia is 53.9% of the welfare level if overall income is equally distributed.

<sup>6</sup> Using this definition, in order to introduce poverty as a multifaceted and multidimensional term, Multidimensional Poverty Index (MPI) came to the academic discussion in 2010.

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subsequent hypothesis is that the calculation result of the relative multidimensional single parameter Gini inequality measure is less than the official data from the traditional Gini coefficient. It means that, high inequality in Colombia is more a matter of income than about distribution in capabilities as knowledge and long and healthy life.

For this purpose, the current paper calculates the multidimensional Gini coefficient for Colombia, considering the three different dimensions of the Human Development Index: health, education and income. In general, the longer life, knowledge and standard of living, the more liberties a person has to choose one or another kind of life to enjoy. Therefore, differences among people in these three dimensions lead to disparities in their possibility to choose between their opportunities. All of them are necessary to live a dignified life.

The calculation with the Gran Encuesta Integrada de Hogares for the Administrative National Department of Statistics of Colombia (DANE) obtains the standard multidimensional Gini index of 0.1572 and in a more bottom sensitive space 0.3632. Both show fewer inequalities across Colombians' households comparing with the one-dimensional measure (0.539).

### 3. THEORETICAL FRAMEWORK OF INEQUALITY CONCEPTS

In the literature exist different indices measuring economic inequality<sup>7</sup>. The one-dimensional Gini coefficient is the most common one. It is a measure of statistical dispersion and inequality, which determines the extent to which the distribution of income or consumption expenditure among individuals or households within an economy deviates from an

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<sup>7</sup> Some measures of inequality are the percentage shares of income or consumption, which describe the share that accrues to subgroups of the population indicated by deciles or quintiles, for example: lowest 10% and highest 10% and the quintiles: lowest 20%, second 20%, third 20%, fourth 20% and highest 20%. Many of them are positive measures, but another normative attracted attention in academic inequality discussions.

equal distribution<sup>8</sup>. It measures the area between a hypothetical line of absolute equality and the Lorenz curve<sup>9</sup>.

According to the two pioneers in the multidimensional inequality welfare concepts Rawls (1973) and Sen (1985), well-being is a multidimensional concept and the determination of dissimilarities has to consider more variables than merely income. Sen believes that a failure in development is more “a failure of basic capabilities to reach certain minimal acceptable levels” (Sen, 1992). This means that the ability to live a dignified life, including being nourished, healthy, etc., summarises a number of key ‘functionings’ and the capabilities to achieve these functionings (Jenkins, 2007).

In the spirit of Sen’s multidimensional approach, various studies determine multidimensional functions and relative multidimensional single parameters as for example the multidimensional Gini inequality measure. Recent studies show that in general multidimensional functions of welfare lead to substantially different results compared to the one-dimensional measures and that sample weighting affects profoundly the empirical outcomes (Anderson, 2005).

Following these pioneers, three researchers highlighted the topic and develop a conceptual and axiomatic framework, whom was based in the axiomatic one-dimensional. Nevertheless, adding more dimensions in calculation, some additional axioms needed to come and others needed to transform<sup>10</sup>. They were Kolm’s (1977) with “Multidimensional egalitarianism”, Atkinson and Bourguignon (1982) with “The comparison of multi-dimensional distributions of economic status” and List (1999) with “Multidimensional Inequality Measurement: A proposal”.

Kolm (1977) investigates the term specific egalitarianism, which implies two properties for the social welfare function: “the allocation of one commodity orders the states of society independently from the

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<sup>8</sup> Income implies the disposable income of a household in a particular year, i.e. the sum of earnings, self-employment, capital income and public cash transfers deducted by social security contributions and income taxes (OECD, 2013).

<sup>9</sup> The Lorenz curve calculates the cumulative percentages of the total amount of received income against the cumulative number of recipients, starting with the poorest individual or household. The Gini coefficient expresses a percentage of the maximum area under the line and varies between 0, which reflects perfect equality and 1, which represents complete inequality (The World Bank, 2013).

<sup>10</sup> The axiomatic set of properties in the one-dimensional scheme is the following: invariance properties (symmetry, population and scale invariance), normalization, dominance (transfer and sensitivity) and subgroup properties.

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allocation of the other ones” and the “preference for a more equal distribution, which in turn implies some anonymity or impartiality assumption”. By analysing both aspects, he proves that multidimensional inequality is a “function of the uniform inequality of a multivariate distribution of goods or attributes across people”.

Atkinson and Bourguignon (1983) investigate a two-dimensional case by means of income and life expectancy. Using results on multivariate stochastic dominance in portfolio theory, they extend to a measurement of inequality. In particular, consider a social welfare function defined over different elements of the vector of goods received by person  $i$  ( $x^i$ ). They take the distribution of per capita incomes and life expectancy across 61 countries and across 1960 -1970. Considering a hypothetical distribution in 1970 and normalizing both variables, they achieve to satisfy the dominance condition<sup>11</sup>.

List (1999) defines two main stages in the previous multidimensional approaches, the dominance criteria that may include only partial orderings and the definition of the inequality index that has to be consistent with the dominance criteria. He constructs an index considering the transformation of multidimensional distributions into welfare-concentration curves. As a result, the multidimensional generalizations satisfy anonymity, normalization, continuity, replication invariance and ratio-scale invariance. Moreover, indices are “sensitive to how uniformly unequal the distribution of goods/attributes across people is” and “to how systematically inequalities in different dimensions are cross-correlated”.

A third wave of assessment guided the research to new propositions, axioms, measures and applications. They were Weymark (2004) with “The normative approach to the measurement of multidimensional inequality”, Decanq and Lugo (2008 with “Measuring inequality of well-being”, Kobust (2011) with Multidimensional inequality indices from ordinal data, and Jordá (2012) with “Multidimensional inequality in the Worldwide wellbeing through generalized entropy indices”.

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<sup>11</sup> The dominance condition describes de idea to show if the investigated parameters are really higher in 1970 than they have been in 1960. Dominance criteria evaluates about the causes to change the measure in a particular direction. The properties associated are transfer sensitivity and the transfer principle.

Weymark (2004) concentrates on an approach that focuses on disparity indices rather than on the dominance criteria. By constructing a social evaluation function to determine an inequality index, he discovers that different population sizes must have different functions due to the properties of decomposability and population replication. After that, uncertainty should be included to work in a more structured environment and to find appropriate axioms.

Decancq and Lugo (2008) derive a multidimensional social evaluation function and the relative multidimensional single parameter Gini inequality measure. They compose a two-step approach, where first calculate an index of individual well-being and then aggregate these indices into a measure of societal well-being. They investigate the multidimensional generalization of the Pigou-Dalton transfer property.

Kobus (2011) evaluates differences in happiness and health, in a two-dimensional measure for US data from 1972-2010. She uses inequality indices invariant to change of scale and copula<sup>12</sup> orderings to capture associations between variables, ensuring that the obtained index is attribute decomposable. This means that the inequality effects must identify differences among disparity in marginals<sup>13</sup> and association between dimensions. Finally, indices that meet normalization, scale independence, continuity and are consistent with inequality ordering build continuous increasing functions of copulas and marginals (Kobus, 2011).

Jordá (2012) uses three different approaches to assess the multidimensional inequality worldwide, considering the Human Development Index for the period between 1980 and 2011. After proposing a dimension-by-dimension approach, he shows a reduction in disparity in three dimensions over the study period. Overall inequality decreased roughly by 40% and the reduction was in the whole three dimensions, income, education and health. Finally by decomposing their measures into regions and within regions components, it is found that

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<sup>12</sup> Copula is the most general measure of association between qualitative variables, which captures those properties of the joint distribution, which are invariant under (...) strictly increasing transformations (Wolff, 1981).

<sup>13</sup> The inequality functions rise with respect to ordering of Copulas and the marginals (continuous increasing functions). The latter rise according to the order presented in the Alkire Foster multidimensional framework. It implies that a better concentration of the marginal distributions imply a better concentration of the joint distribution (Kobus, 2011).



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the main fall of disparity is produced due to the decrease in the within component (Jordá, 2012).

### 4. METHODOLOGY

This work bases its methodology on the normative procedure to derive a measure of well-being inequality by determining a multidimensional social evaluation function. The paper leans mainly onto the recent work of Decancq and Lugo on normative multidimensional social evaluation function and the relative multidimensional single parameter Gini inequality measures<sup>14</sup>.

They determine the multidimensional function with two distinct steps. First, they derive an objective well-being index in order to summarize the outcomes of every individual across the dimensions of well-being. This objective well-being index reflects the preferences of the society over the different dimensions. In the second step, they aggregate the well-being indices across individuals to find a measure of the society as a whole, depending on the position and rank of the individual in the total distribution (Decancq, 2008).

For their first step to derive an individual well-being index that aggregates individual's outcomes across all well-being dimensions, Decancq and Lugo proceed like the following.

With two sets, being  $\mathbf{N}$  the set of well-being and  $\mathbf{M}$  the set of the dimensions, the element  $x_i^j$  of the distribution matrix  $n \times m$  ( $\mathbf{X}$ ) represents the outcomes of the individual  $i$  on dimension  $j$ . Following that logic,  $\mathbf{x}_i$  is a row vector of the distribution matrix  $\mathbf{X}$  where are the outcomes of an individual, and  $\mathbf{x}^j$  is the column vector where is the distribution of the  $j$ -th dimension of well-being.

In their paper they determine a well-being function  $\mathbf{S}$  by having the set  $\mathbf{V}$  as a set of admissible outcome vectors  $\mathbf{x}_i$ , which represents a binary relation on the set. The well-being function  $\mathbf{S}$  “aggregates outcomes across the multiple dimensions of well-being and captures

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<sup>14</sup> The reason of following specifically Decancq and Lugo framework is because their paper fit with the aim of compare the traditional one-dimensional inequality measure (Gini coefficient) in Colombia with the multidimensional one. Furthermore, its paper analyses and presents a solution to the Pigou-Dalton principle in a multidimensional space.

different value judgments regarding trade-offs between dimensions and admissible transformation among others” (Lugo, 2005).

The value judgments are represented by the following properties which together represent a six set for the measurement problem of well-being: Ordering-Property (ORD), Continuity-Property (CONT), Monotonicity-Property (MON), Dimension Separability-Property (DSEP), Strong Homotheticity-Property (SHOM) and Weak Homotheticity-Property (WHOM).

Taking a closer look on the six properties, the ORD-Property shows that the well-being function is reflexive, complete and transitive on the set  $\mathbf{V}$ . The CONT-Property ensures that  $\mathbf{S}$  is continuous and so not oversensitive to small changes in the outcomes. The MON-Property shows that all dimensions are desirable. The DSEP-Property, separability across dimensions, explains that the exact level of the common outcome is not decisive to the well-being relation when two outcome vectors show the same outcome for a specific dimension. The SHOM-Property represents that rescaling dimensions is possible without changing the well-being relation. With strong homotheticity, the well-being order does not change by rescaling, while the weak homotheticity (WHOM-Property) leads to a proportional change by rescaling.

With the six properties above, some propositions follows. Proposition 1 says that a well-being relation on  $\mathbf{V}$ :

a) Satisfies the ORD; CONT; MON and SHOM-Properties only if the well-being relation is represented by the well-being function with  $w_j > 0$  for all  $j \in M$  and  $\sum_{j=1}^m w_j = 1$

$$S(x_i) = \prod_{j=1}^m (x_i^j)^{w_j} \quad (1)$$

b) Satisfies the ORD; CONT; MON; DSEP; WHOM-Properties only if it is represented by the well-being function next with  $w_j > 0$  for all  $j \in M$  and  $\sum_{j=1}^m w_j = 1$ .

$$S(x_i) = \left( \sum_{j=1}^m w_j (x_i^j)^\beta \right)^{\frac{1}{\beta}} \quad (2)$$

Choosing SHOM the Cob-Douglas well-being function is necessary because it has unit elasticity of substitution in order to set unchanged one of the properties. On the contrary, by using WHOM, a constant

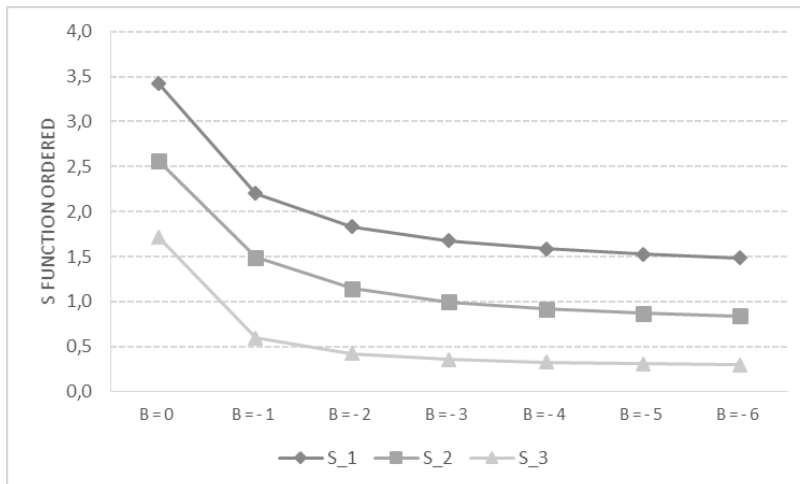
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elasticity of substitution (CES) function of proposition 1b) can be obtained. The parameter  $\beta$  and the positive weights  $w_j$  together determine the trade-offs between the different dimensions, where  $\beta$  is the elasticity of substitution between the different dimensions of well-being and  $w_j$  the relative importance of the particular dimensions.

In order to measure disparity in well-being, is important to take into account the inequality between the cardinal labels. This cardinalization uses the equally distributed equivalent outcome ( $s_i$ ). “It is the outcome level  $s_i$  that obtained equally by individual  $i$  in all dimensions, leads to the same level of well-being as the observed outcome vector”.

Proposition 1 reflects the m-dimensional iso-wellbeing curves. As the graph describes, every curve represents the well-being function for every person in the group of analysis as beta is changing its value. The lowest curve implies lowers outcomes for every individual and the higher the opposite.

Graph 1. M-dimensional Iso-wellbeing Curves



Source: Own elaboration

In the second step, Decancq and Lugo characterize a class of social evaluation functions  $W$  to aggregate well-being indices across individuals, like the following.

The set of properties consists on the Ordering-Property (ORDR), the Continuity-Property (CONTR), the Monotonicity-Property (MONR), the Individual Separability-Property (ISEPR) and the Linear Invariance-Property (LINVR).

These five properties determine proposition 2: A social evaluation relation ( $R^n$ ) satisfies the ORDR; CONTR; MONR; ISEPR and LINVR -Properties only if the social evaluation relation corresponds to  $W(S_x) = \sum_{i=1}^n a_i S(x_{|i|})$  with  $a_i > 0$  for all  $i$  in  $N$  and  $\sum_{i=1}^n a_i = 1$ . For  $n = 1$  the formula represents the one-dimensional Gini coefficient. The vector of non-negative welfare weights  $a_i$  includes the weights that depend on the rank of one individual compared to the whole society. Furthermore, the Principle of Population-Property (POPR) and the Restricted Aggregation-Property (RAR) permit to order the well-being vectors of different length.

Proposition 3 says that a social evaluation relation  $R^n$  on  $S$  satisfies the ORDR; CONTR; MONR; ISEPR; LINVR; POPR and RAR- Properties, if the social evaluation relation is determined by

$$W(S_x|\delta) = \sum_{i=1}^n \left[ \left(\frac{i}{n}\right)^\delta - \left(\frac{i-1}{n}\right)^\delta \right] S(x_{|i|}) \text{ with } \delta > 0 \quad (3)$$

In the social evaluation function, the parameter  $\delta$  represents the bottom sensitivity by capturing the sequence of weights and the associated value judgements<sup>15</sup>. If  $\delta = 1$  the social evaluation function represents the utilitarian and traditional not weighted average of the well-being indices. When increasing  $\delta$  the bottom distributions have more weight until the Rawlsian social evaluation function ( $\delta = +\infty$ ). On the other side, a decreasing  $\delta$  the best off individuals get more weight. The traditional Gini index is represented in  $\delta = 2$ .

Before finally deriving the multidimensional single Gini inequality measure, the distributional concerns focus on the multidimensional transformation of the Pigou-Dalton transfer principle and the effect of changes in correlation between dimensions<sup>16</sup>.

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<sup>15</sup> The paper assumes that every weight of every dimension will represent people value judgement. Therefore, bigger delta enhances the larger importance of lower values of the well-being functions.

<sup>16</sup> Afterwards, the authors generalize the Pigou-Dalton transfer principle, which says that shifting income from a poor person to a richer one leads to a decrease in social welfare, and they investigate the effect of imposing it in multidimensional inequalities.

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Bringing all together, sets of properties and propositions, Decancq and Lugo conceive the following compound social evaluation function:

$$W(X|\delta, \beta, w) = \sum_{i=1}^n \left[ \left( \frac{i}{n} \right)^\delta - \left( \frac{i-1}{n} \right)^\delta \right] \left( \sum_{j=1}^m w_j (x_{(i)}^j)^\beta \right)^{\frac{1}{\beta}} \quad (4)$$

It represents the compound multidimensional social evaluation relation, where  $\{i\}$  stands for the position of the well-being in the distribution of well-being indices,  $\delta$  determines the bottom sensitivity,  $\beta$  interprets the degree of substitutability between dimensions and  $w$  stands for relative importance of outcomes in the aggregation.

In the social evaluation relation, the Uniform Majorization-Property (UM) limits the relation for  $\beta < 1$  and  $\delta > 1$ . However, it is not sensitive to correlations between dimensions<sup>17</sup> and cannot satisfy the property of Correlation Increasing Majorization-Property (CIM).

As last step, Decancq and Lugo define the multidimensional S-Gini inequality index applying the compound social evaluation relation to the scalar of a relative inequality measure  $I_R(X)$ , where  $\beta < 1$  and  $\delta > 1$ :

$$I_R(X) = 1 - \frac{\sum_{i=1}^n \left[ \left( \frac{i}{n} \right)^\delta - \left( \frac{i-1}{n} \right)^\delta \right] \left( \sum_{j=1}^m w_j (x_{(i)}^j)^\beta \right)^{\frac{1}{\beta}}}{\left( \sum_{j=1}^m w_j \mu(x^j) \right)^{\frac{1}{\beta}}} \quad (5)$$

### 5. DIMENSIONS, INDICATORS AND DATA FOR THE MULTIDIMENSIONAL INEQUALITY CALCULATION

In order to derive the multidimensional Gini index, this work follows the different Human Development Reports since 1990 considering the three capabilities (1) a long and healthy life, (2) knowledge and education and (3) a good standard of living as determinants in order to enjoy a dignified life<sup>18</sup>. Adopting this concept and supporting the idea

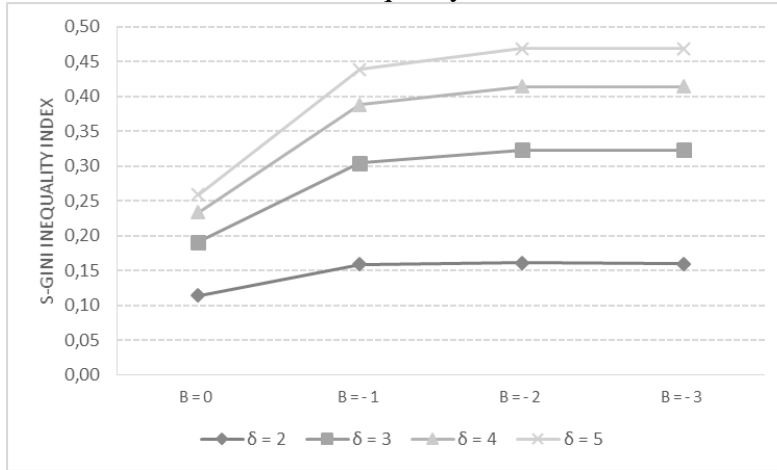
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<sup>17</sup> Correlation means that if having two matrixes with the same marginal distribution, the one with lower correlation between the dimensions should be preferred.

<sup>18</sup> The first HDR of 1990 introduced the Human Development Index (HDI) to measure development combining the three dimensions health, education and living standard and defining them as the basic and substantial dimensions in order to be able to live a dignified life. In order to measure the three dimensions, the HDR uses the four indicators life expectancy at birth, mean years of schooling, expected years of

that differences between people lead to dissimilarities in their liberty of choices. The current calculation of multidimensional inequality measure takes all of them into account.

Graph 2.  
Multidimensional S-Gini Inequality Index Curves



Source: Own elaboration

Health, as first dimension, is a necessary condition to live the life someone wants to live. A sick person without or with limited access to the health system has limited freedom and cannot live the life he or she wishes to. The indicator “affiliation to the social security and health system” is a representative indicator for this dimension. It includes access to the Health Compulsory Plan (subsidiary or not), which includes medicines and medical procedures, in case of illness, family planning, sterilization and disabilities, between others<sup>19</sup>.

In the education dimension, knowledge is a necessary concept to guarantee possibilities of choice. If someone has absolutely no education and / or literacy this person is imposed to many restrictions and has no access to a productive life in modern society. The person has

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schooling and gross national income per capita defining specific goalposts for every dimension (UNDP, 2013).

<sup>19</sup> The Health Compulsory Plan in Colombia is explicit in the Agreement 30/2012 of the National Health Regulatory Commission (<http://www.minsalud.gov.co/salud/POS/Paginas/Acuerdo%2029%20de%202011.aspx>). In this variable, 1 stands for having no access and 2 for having access to the health and security system.

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limited options of living and choosing. In order to measure, the calculation uses the indicator “mean years of schooling”<sup>20</sup> of all household members older than 15 years.

Finally, a quite equally distributed income is necessary to guarantee equality across individuals. If there are big differences of income in a society, then some groups have the possibility to live the life they want to and decide over others while others have less freedom of choice. Income inequalities can lead to social problems, dissatisfaction of the citizens and poverty. Therefore, the calculation uses the variable “monthly labour income of employed persons”<sup>21</sup>.

Table A  
Dimensions and indicators of the multidimensional calculation

The multidimensional Gini coefficient			
Liberty / Capability Dimension	Long and healthy life Health	Knowledge and education Education	Level of dignified life Income
Indicators for Colombia	Affiliation to the social security and health system	Years of schooling for household members >=15 years	Monthly labour income of employed persons

Source: Own elaboration

The sample comes from the “Gran Encuesta Integrada de Hogares 2012”, a series of nearly annual, nationally representative surveys designed to monitor employment conditions and general characteristics of the Colombian population.

As first step, the paper constructs a complex matrix aggregating the subdivided data sample of urban, rural and other areas to one complete sample in order to have a look on the overall multidimensional inequality of Colombia. Furthermore, the matrix includes the three indicators for the respective dimensions of well-being: affiliation to the health system, years of schooling and monthly income.

<sup>20</sup> It is the aggregate of the highest diploma and the highest education level for people older than 15 years.

<sup>21</sup> The monthly-obtained wage are salaries plus bonus, overtime hours and further incomes. The constructed monthly income of employed persons is widely used in literature as indicator for material standard of living.

As second step, the approach only considers households whose data is available for the three dimensions, leaving a sample about 194.382 households<sup>22</sup>.

In literature and practice, a variety of methods exists to handle missing data, which arise in almost all serious statistical analyses. Statistical procedures, on how to treat missing values, include methods of removing variables or cases with missing data and fill or impute missing values. Within these techniques, important methods are case deletion, available case analysis, mean imputation, median imputation, KNN imputation, etc. Classical regressions often exclude all cases with any missing inputs, also known as complete-case analysis. This can limit the amount of information available in the analysis, especially if the model includes many inputs with potential missingness (Acuña, 2004 and Rubin, 2002).

This multidimensional calculation partially uses the case deletion in addition to the mean imputation method. The approach deletes Colombian households with missing values in more or equal to two dimensions. In order to not reduce the household sample dramatically, it further assumes that the remaining missing values across the household's members adopt themselves to the household mean in the dimensions education and income. As the health performance of one household ordering through its maximum, missing values across household members in this dimension do not matter. If one household member has no access to the health and security system the whole households is without access.

Finally, the paper applies the complete-case analysis to the assembled households by deleting households with missing values in at least one of the three dimensions. Considering these deletion, it was 35 household omitted, due to missing values in the health dimension and only twelve households due to incomplete information in education, while more than 30,000 households did not declare their income.

With the previous deletion methods, the paper uses only the households for which complete data is available for all three dimensions. Additionally, households with zero values in one of the three dimensions do not count in order to guarantee a correct application of the Gini social evaluation function.

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<sup>22</sup> Comparing the sample of about 189,782 Colombian households to the quantity used in Decancq and Lugo for Russian example, they took 6,000 individuals in consideration.



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### 6. TENDENCIES IN EDUCATION, HEALTH AND INCOME

Even though Colombia is part of the upper-middle income countries, it faces difficulties on the labour market, sticking out the high unemployment and vulnerable employment rate as striking features (The World Bank, 2012). Colombian economy has a GNI per capita of U\$ 9.990 in 2012, up to the worldwide average U\$ 9.096. Unemployment of labour force was 9.6% and exceeded the worldwide of 5.9% and the Latin American & Caribbean of 7.7%. Within the employed persons the vulnerable, the share of unpaid family workers and own-account, is 49%, quite high compared to 31% in whole Latin America & the Caribbean.

Using international extreme poverty standards, Colombia had in 2010 8.2%, quite low compared with the worldwide data 20.6% of people living with less than 1.25 USD per day. Nevertheless, the Colombian poverty rates are higher than the poverty average of Latin America & the Caribbean (The World Bank, 2012).

In the health dimension, since National Constitution of 1991, the Health System is a collective risk fund where formal, informal and no workers with their families can receive benefits in medicines and procedures in health. Colombia increased smoothly the coverage since then, and 2013 has reached 91.6% of people affiliated to the Insurance Health System. Between affiliated people, 48.11% are subsidized and 42.76% non-subsidized. Adding up health insurance of public professors and army forces (Exception Regimes), the percentage increase to 96.4%. It means that no affiliates are 3.6% of Colombia's population. Additionally to affiliation, in life expectancy, access to improved water resources, sanitation facilities and maternal mortality, Colombia exceed the worldwide and almost equal the Latin American average (The World Bank, 2012 & 2013).

In the education dimension, Colombia has quite a good performance, goes beyond the worldwide and Latin American countries. Gross National Coverage in primary and secondary education increased year by year to reach in 2012 to 100.76% and Net National Coverage to 88.31%. Primary completion rate counts with a share of 112%<sup>23</sup> and

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<sup>23</sup> “The Primary completion rate counts the number of new entrants (enrolments minus repeaters) in the last grade of primary education, regardless of age, divided by the

literacy<sup>24</sup> is about 98%. This governmental effort transformed the average years of education from 8.5 in 2002 to almost 10 in 2013 in people between 15 and 24 years.

In terms of inequality, a positive tendency in Colombia happened over the last decade. Between 2002 and 2012, the Income Gini coefficient on national level decreased from 0.572 to 0.539, which represents a total reduction of 3.3%. In addition, per capita income in poorer groups grew six times higher comparing with the wealthier groups. Between 2010 and 2012, the growth of the income per capita in the first quintile was about 13.2% while the per capita income in the last quintile just grew by 2.3%.

According to the DANE survey used, the average values of the three indicators show that households in Colombia have a quite reliable access to the health system; education performance with an average of 9 years (15 years older) of schooling and the average income per household is 789.739 Colombian pesos, which correspond to approximately.

That mean income corresponds to approximately 5.096 USD per year in PPP\$ or approximately 8.300 USD of the average wage reported by the International Labour Organization<sup>25</sup>. Value that is lower than the GDI per capita of 9.990 U\$ per year in 2013, showing restrictions to spread the income earned by growth in the whole economy.

Dividing the sample survey by quintiles, it shows that as income is ascending, people enjoy a better state of wellbeing. The average school years and the monthly average income increase as quintiles do. While the first quintile in average only have 6.65 years of schooling, the last quintile almost doubles it (12.63 years). Similarly are the differences in income, on average the first quintile earns just 8% of the last. Only health dimension order changes by passing through the quintiles.

Within Latin America, Colombia is one of the most unequal countries. Comparing the current inequality index with the world Colombia occupies rank 11 in inequality and the fourth in the region. The percentage share of income or consumption confirms the later. The lowest 10% possesses 0.9% of the total income while the highest 10%,

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population at the entrance age for the last grade of primary education” (The World Bank, 2013).

<sup>24</sup> Literacy rate describes the percentage of the population ages 15–24 that can read and write a short simple statement about their everyday life.

<sup>25</sup> Differences among values are because of the taxes effect.

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owns around 44.4%. In addition, taking the distribution in quintiles, the first possesses 3%, the second 6.8%, the third owns 11.2%, the fourth quintile gets 18.8% of the total income and finally the wealthiest 20% of Colombia own 60.2% (The World Bank, 2012).

Table B  
Quintile matrix of Colombia

	Health	Education	Income US year	Income \$ month
	1,75	6,65	1.049	162.531
X (Quintile) =	1,72	7,53	2.489	385.698
	1,75	8,58	3.631	562.718
	1,78	9,72	5.006	775.791
	1,86	12,63	12.859	1.992.993
Total survey	1,77	9,17	5.096	789.739

Source: Own elaboration

According to the Organisation for Economic Cooperation and Development (OECD), reasons for the still high inequality in Colombia originate from the labour market and lie, between others, on the high unemployment rate, a pervasive informal sector and wide wage dispersion with evidence of the large education premium having higher education. In order to reduce that differences policies needs to modify the labour market by improving the labour supply and productivity, and second, reducing the informal sector and wages dispersion among formal jobs (Joumard, 2013).

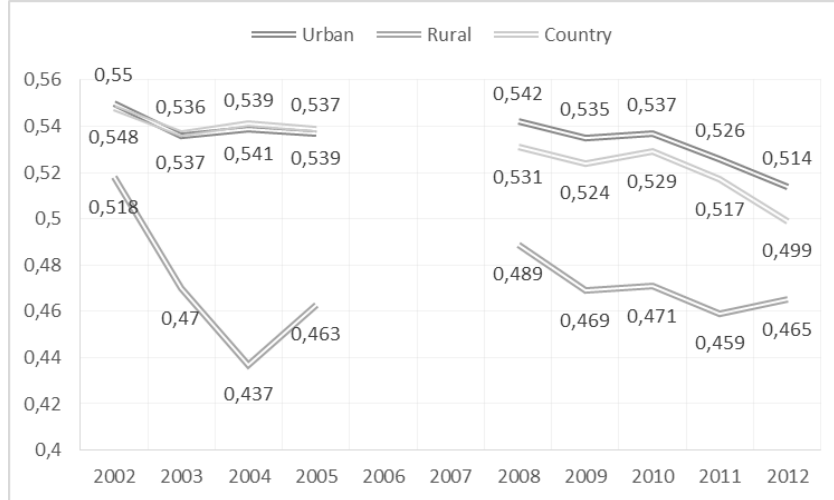
One of the main reasons for the remaining high inequality are differences between areas (Urban<sup>26</sup>/rural<sup>27</sup>). Urban areas are capturing a massive amount of people that come from the rural. Many causes leads the migratory process, conflict, economical issues, among others. Rural areas normally are less unequal than urban, but poorer. Furthermore, differences in districts are important as well. As an example,

<sup>26</sup> Urban areas, in terms of DANE, are sets of edifications and blocks, delimited by streets mainly. In general, counts with essential public goods as connected services of water and sewer, electricity, hospitals and schools, between others. Cities and municipal heads are part of the urban area.

<sup>27</sup> Rural areas are a wide land with farming activities. It does not count with nomenclatures or streets, neither public services nor facilities.

Bucaramanga and Cúcuta had a Gini coefficient of about 0.4 while in Cali was 0.515 (DANE, 2012).

Graph 3. Gini coefficient for Colombia, urban and rural areas, 2002 - 2012



Source: Own elaboration using DANE - Encuesta Continua de Hogares (2002-2006) y Gran Encuesta Integrada de Hogares (2008-2012)

## 7. RESULTS IN MULTIDIMENSIONAL GINI INDEX FOR COLOMBIA

In order to obtain the multidimensional Gini indices, the calculus uses the previously described two stages process of Decancq and Lugo. Within this calculation, the well-being measure uses equal positive dimension weights ( $w_j = 1/3$ ) summing up to 1 and therefore reflecting the equally distributed importance of the different dimensions.

The result shows the evolution of the S – Gini index for Colombia  $I_R(X)$  depending on  $\beta$ , the respective elasticity of substitutability between the dimensions, and  $\delta$ , the bottom sensitivity. The table describes that, whether  $\beta$  is  $\delta$  increasing, it leads to  $I_R(X)$  up.

Using a Cobb-Douglas wellbeing function to measure, Decancq and Lugo recommend to select a  $\beta = 0$ . It is useful, in order to consider a unitary elasticity of substitution, the satisfaction of the Strong Homotheticity Property SHOM. Robust results with strong-ratio scale invariance, which guaranty first, that curves don't cross each other, second, wellbeing order do not change by rescaling (Decancq and Lugo, 2009) and third, conserve one characteristic of homogeneous functions:

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implicit interchanges between variables just depend on quotient of these variables, not on their absolute levels (Nicholson, 2007).

Focusing on the first column of the table, with  $\delta = 2$  the multidimensional calculation obtains the standard Gini index of 0.1572, according to Decanq and Lugo. Increasing the weight given to the worst-off individuals, with  $\delta = 5$ , the multidimensional Gini index for Colombia climbs to 0.3632. In both cases, the results are significantly lesser than the one-dimensional and traditional Gini index of 0.539. The latter confirms that income inequalities in Colombia are deeper than inequality in other capabilities.

Table C  
Multidimensional indices of Colombia, depending on  $\beta$  and  $\delta$

	For $\delta =$	$\beta = 0$	$\beta = -1$	$\beta = -2$
$I_R(X) =$	2	0.1572	0.2276	0.2304
	3	0.2593	0.3655	0.3861
	4	0.3205	0.4476	0.4762
	5	0.3632	0.5048	0.5379

Source: Own elaboration

Computing the multidimensional Gini using income quintiles, results indicate that for  $\delta = 2$  the multidimensional calculation obtains the standard Gini index of 0.1442 and increasing the bottom sensitivity, with  $\delta = 5$ , the multidimensional Gini index for Colombia is 0.3093. Those values are lower than the previously obtained.

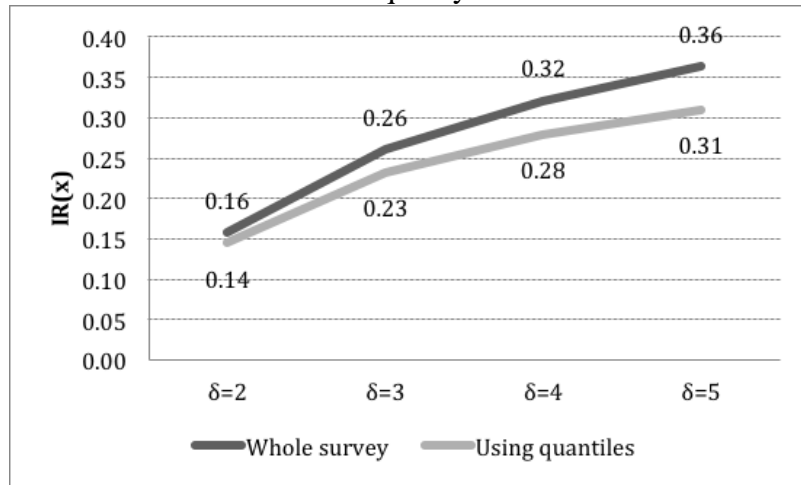
In the same way, computing quintiles differences is interesting to evaluate the value of the multidimensional measure comparing one quintile with each other. If it just uses  $\delta = 2$  and  $\beta = 0$ , the measure considering the first two quintiles is 0.0683, 0.0831 among the upper two quintiles and 0.1339 in the two extreme quintiles, the first and the last one.

One aspect arises on these results is that the way to correct income inequalities relies on changing the capabilities distribution

first<sup>28</sup>. Let's say, following OECD recommendation of that policies needs to develop labour market by improving labour supply and productivity, the space where Colombian national policies needs to focus is on capabilities, as apparently it is happening.

Current investment in education and health are fundamental capabilities to enhance people productivity. Consistent with the latter results, inequalities in Colombia is not a matter of coverage in health, what suggest the need to analyse in detail the quality of attention. In spite of health, reducing education inequality is fundamental in order to reduce other kind of inequalities, specially the income one.

Graph 4.  
Multidimensional S-Gini Inequality Index for Colombia 2012



Source: Own elaboration

Now, the next question is related with the differences between regions. In order to analyse it, the national multidimensional inequality measure needs to be divided in subgroups for Colombian regions. Is important to have in mind that traditional Gini coefficient don't satisfies perfectly subgroup properties, specifically subgroup consistency, in the same way that the multidimensional Gini index certainly can't satisfy.

The results are in the next table for every delta. The table shows the multidimensional inequality index for deltas 2 to 5, and betas from 0

<sup>28</sup> Income dispersion is caused, among others, by the high unemployment rate, a pervasive informal sector and the wide wage dispersion with evidence of the large education premium having higher education.

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to -2, for each delta. Using delta 2 and beta 0, as well as before, regions in Colombia are between 0,137 (Atlántico) and 0,171 (Chocó).

Those results show the differences in opportunities for Colombian when born in one region or another. The top five regions with less inequality are Atlántico, Risaralda, Bolívar, Caldas and Cundinamarca. On the other hand, the bottoms five are Chocó, Nariño, Cauca, La Guajira and Huila. But complementarily, the results show that the more income inequality doesn't mean the more multidimensional inequality.

### 8. CONCLUDING REMARKS

Attached on the capability approach and the diversity of human needs this paper shows that inequality is not just a matter of income but wellbeing depends on the capability to achieve several valuable capabilities as long and healthy life and knowledge. Persons who face multiple deprivations are normally worse off than persons with only few of them are.

Income inequalities in Colombia are deeper than inequality in other capabilities. Using a Cobb-Douglas function, the multidimensional calculation obtains the standard Gini index of 0.1572 when  $\delta = 2$  and of 0.3632 with  $\delta = 5$ . In both cases, the results are significantly lesser than the one-dimensional and traditional Gini index of 0.539. Calculating with quintiles, the results are similar.

Breaking results by regions, they show the differences in opportunities for Colombian when born in one region or another. Using delta 2 and beta 0, regions in Colombia are between 0,137 (Atlántico) and 0,171 (Chocó).

Besides the high-income inequality of Colombia, education also has significant negative effects on the multidimensional inequality of Colombia, which have remarkable influence on the labour market. Differences in education are caused by an unequal access to the education system across household and regions in connection with the wide wage dispersion. While wealthier households have a higher mean of schooling and university years, poorer households attend school for less time.

Table 1 Multidimensional S-Gini Inequality Index for Colombia by regions 2012

Regions	Delta=2			Delta=3			Delta=4			Delta=5		
	$\beta=0$	$\beta=-1$	$\beta=-2$	$\beta=0$	$\beta=-1$	$\beta=-2$	$\beta=0$	$\beta=-1$	$\beta=-2$	$\beta=0$	$\beta=-1$	$\beta=-2$
Atlántico	0,137	0,201	0,206	0,221	0,318	0,338	0,273	0,392	0,419	0,310	0,445	0,477
Risaralda	0,148	0,209	0,212	0,244	0,334	0,354	0,302	0,411	0,438	0,344	0,466	0,497
Bolívar	0,149	0,219	0,225	0,242	0,344	0,366	0,298	0,419	0,448	0,337	0,471	0,505
Caldas	0,149	0,216	0,225	0,243	0,338	0,361	0,300	0,412	0,442	0,340	0,464	0,498
Cundinamarca	0,150	0,208	0,199	0,254	0,354	0,365	0,318	0,444	0,466	0,364	0,508	0,536
Santander	0,151	0,219	0,232	0,247	0,337	0,364	0,305	0,409	0,442	0,346	0,459	0,496
Sucre	0,152	0,219	0,223	0,247	0,343	0,364	0,304	0,417	0,446	0,344	0,469	0,502
Bogotá D.C.	0,152	0,221	0,241	0,240	0,333	0,364	0,292	0,399	0,435	0,329	0,446	0,485
Meta	0,152	0,217	0,226	0,249	0,339	0,362	0,307	0,413	0,443	0,348	0,466	0,500
Cesar	0,152	0,226	0,232	0,249	0,355	0,378	0,306	0,432	0,463	0,346	0,486	0,521
Valle del Cauca	0,153	0,215	0,215	0,253	0,353	0,369	0,315	0,438	0,462	0,359	0,498	0,527
Magdalena	0,153	0,221	0,222	0,251	0,356	0,375	0,309	0,437	0,464	0,350	0,494	0,525
Antioquia	0,153	0,219	0,223	0,255	0,357	0,378	0,318	0,442	0,470	0,362	0,502	0,534
Boyacá	0,157	0,230	0,238	0,264	0,371	0,397	0,328	0,453	0,487	0,372	0,509	0,547
Nte Santander	0,157	0,224	0,228	0,256	0,352	0,371	0,315	0,428	0,456	0,356	0,482	0,515
Tolima	0,157	0,225	0,224	0,264	0,370	0,387	0,328	0,457	0,482	0,373	0,517	0,548
Caquetá	0,160	0,231	0,240	0,257	0,353	0,378	0,313	0,423	0,456	0,351	0,471	0,508
Quindío	0,160	0,225	0,218	0,267	0,380	0,394	0,331	0,472	0,495	0,375	0,536	0,563
Córdoba	0,161	0,233	0,229	0,265	0,383	0,400	0,327	0,470	0,496	0,370	0,530	0,561
Huila	0,162	0,239	0,245	0,268	0,380	0,405	0,330	0,463	0,496	0,373	0,519	0,557
La Guajira	0,164	0,238	0,236	0,276	0,391	0,409	0,343	0,478	0,505	0,388	0,538	0,569
Cauca	0,168	0,244	0,240	0,280	0,399	0,417	0,345	0,487	0,513	0,390	0,545	0,576
Nariño	0,169	0,251	0,246	0,276	0,405	0,424	0,337	0,490	0,518	0,378	0,545	0,577
Chocó	0,171	0,251	0,245	0,286	0,414	0,431	0,353	0,505	0,532	0,398	0,566	0,597

Source: Own elaboration



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